Audio file

118921-Science&SocietyTheConceptOTruthInMathematicByAmelieBerger.mp3

Transcript

00:00:02 Speaker 1

So today I'm going to talk about the concept of truth in mathematics.

00:00:07 Speaker 1

And I was inspired to make this talk because what I really liked about mathematics when I was at school was that it was black and white.

00:00:16 Speaker 1

It wasn't grey, yes.

00:00:18 Speaker 1

English poetry, it was grey.

00:00:20 Speaker 1

What does the poet mean when he says this and this and this?

00:00:24 Speaker 1

Don't know.

00:00:24 Speaker 1

Nobody knows the answer yet.

00:00:26 Speaker 1

Whereas maths, it was right or it was wrong.

00:00:29 Speaker 1

And it is the same.

00:00:30 Speaker 1

Doesn't matter which country you come from, doesn't matter which you speak.

00:00:33 Speaker 1

So I really like this idea that things are either right or wrong.

00:00:37 Speaker 1

So what I would like to talk about in a little bit more detail is if something's right in mathematics, what exactly is truth in mathematics?

00:00:47 Speaker 1

And if

00:00:49 Speaker 1

somebody had to answer this question.

00:00:51 Speaker 1

What does it mean if something's true in mathematics?

00:00:54 Speaker 1

Then I think an answer that you're quite likely to get is that it's something that you can prove.

00:01:00 Speaker 1

Yes, you can prove your result.

00:01:02 Speaker 1

And this is indeed part of the answer, that you can get to your results by using logic, logical argument.

00:01:09 Speaker 1

That's all that proof is, right?

00:01:10 Speaker 1

It's just logical argument.

00:01:12 Speaker 1

From one step to the next step, we use logic.

00:01:14 Speaker 1

So the first part of today's

00:01:16 Speaker 1

talk, I would like to speak a little bit more about logic, which is the basis of proof in mathematics.

00:01:25 Speaker 1

So first of all, the history of the subject of logic.

00:01:28 Speaker 1

Now, all societies across the world would have used logic forever, right?

00:01:32 Speaker 1

But logic was first studied formally in ancient Greece, where Aristotle studied the idea of syllogisms.

00:01:45 Speaker 1

What are syllogisms?

00:01:47 Speaker 1

Syllogisms are very short, basic examples of valid logical reasoning.

00:01:54 Speaker 1

Let's give an example of a syllogism.

00:01:57 Speaker 1

So any syllogism comes with some facts that you accept as being true, and then a conclusion you can draw from it.

00:02:04 Speaker 1

So here's an example.

00:02:07 Speaker 1

Tonight, we are going to have spaghetti or pizza for dinner.

00:02:10 Speaker 1

This is a fact.

00:02:11 Speaker 1

Let's accept it.

00:02:12 Speaker 1

Yes, that is what we're going to have for dinner tonight.

00:02:14 Speaker 1

Now, here comes another fact for you.

00:02:17 Speaker 1

Tonight, we cannot have pizza.

00:02:19 Speaker 1

I don't know.

00:02:19 Speaker 1

The oven is broken.

00:02:20 Speaker 1

There's no cheese.

00:02:21 Speaker 1

For some reason, we cannot have pizza.

00:02:23 Speaker 1

All right, so what is your logical conclusion from this?

00:02:28 Speaker 1

We're going to have spaghetti tonight, right?

00:02:29 Speaker 1

So this is obvious.

00:02:30 Speaker 1

So this is an example of a syllogism.

00:02:34 Speaker 1

So A syllogism starts with some facts and then what logically follows from that.

00:02:40 Speaker 1

Now there are all different types of syllogisms and what Aristotle tried to do was to write them all down, all the various types, put them into categories and sort of list them all.

00:02:51 Speaker 1

So this type of arguments are really basic valid reasoning, right?

00:02:56 Speaker 1

It's called a syllogism, introduced, first studied by Aristotle, 300.

00:03:01 Speaker 1

842322 BC.

00:03:05 Speaker 1

OK.

00:03:06 Speaker 1

What is a syllogism?

00:03:07 Speaker 1

It's just a very short, basic argument.

00:03:12 Speaker 1

OK.

00:03:16 Speaker 1

And all syllogisms sort of come like this.

00:03:18 Speaker 1

You've given some facts that you are assuming to be true.

00:03:22 Speaker 1

I've got a pointer.

00:03:24 Speaker 1

I've got a laser pointer, yes.

00:03:26 Speaker 1

So you're given some facts that you assume to be true, these two at the top.

00:03:30 Speaker 1

These are called the premises.

00:03:33 Speaker 1

Yes, premises, stuff that we accept as being true.

00:03:35 Speaker 1

So I accept that we're going to have spaghetti or pizza for dinner.

00:03:38 Speaker 1

I accept that we cannot have pizza.

00:03:40 Speaker 1

And then from that, we draw a conclusion, right?

00:03:43 Speaker 1

So a little syllabism looks like that.

00:03:47 Speaker 1

OK.

00:03:48 Speaker 1

All right.

00:03:49 Speaker 1

Good.

00:03:50 Speaker 1

So language

00:03:53 Speaker 1

is not very important, actually, in these arguments.

00:03:57 Speaker 1

Yes, would you agree?

00:03:58 Speaker 1

You have done arguments like the previous one many times.

00:04:01 Speaker 1

Tonight I must study physics or maths.

00:04:04 Speaker 1

Oh, my friend has got my physics notes.

00:04:07 Speaker 1

Oh, well, then, tonight I must study maths.

00:04:08 Speaker 1

Yes, it is the same thing.

00:04:10 Speaker 1

It is the same argument.

00:04:11 Speaker 1

Also, in English, words are not very precise.

00:04:16 Speaker 1

And in all languages, words are not very precise, right?

00:04:19 Speaker 1

For example, this word is in English, yes?

00:04:22 Speaker 1

Donald Trump is the President of the United States.

00:04:26 Speaker 1

It means that the two things are exactly the same.

00:04:28 Speaker 1

Donald Trump, President of the United States.

00:04:29 Speaker 1

They are the same person, yes?

00:04:31 Speaker 1

But he is English.

00:04:33 Speaker 1

Is he equal to all English people?

00:04:35 Speaker 1

No, he isn't, right?

00:04:36 Speaker 1

But in our heads, immediately we can separate these things out.

00:04:40 Speaker 1

But language is

00:04:44 Speaker 1

open to misinterpretation.

00:04:46 Speaker 1

So what we do for these synergisms is we want to move away from language.

00:04:51 Speaker 1

We want to be more precise, and we want to cover all possibilities.

00:04:55 Speaker 1

So mathematics has formalized these little valid reasoning arguments into symbols.

00:05:03 Speaker 1

And I'm going to show you an example of what the symbols look like.

00:05:07 Speaker 1

You may have seen it before.

00:05:08 Speaker 1

If you have, great.

00:05:09 Speaker 1

If you haven't, it doesn't matter at all, right?

00:05:11 Speaker 1

So this one

00:05:14 Speaker 1

It's saying that one of two things is true, either spaghetti or pizza is true, and then we cannot have pizza.

00:05:20 Speaker 1

And it's written in symbols in this kind of a way.

00:05:23 Speaker 1

You've seen this sort of thing before?

00:05:26 Speaker 1

Almost certainly, yes.

00:05:27 Speaker 1

S or P is true.

00:05:29 Speaker 1

P is not true.

00:05:31 Speaker 1

Conclusion, S is true, right?

00:05:33 Speaker 1

So it's in symbols like this, yes.

00:05:35 Speaker 1

Do you need to know this?

00:05:36 Speaker 1

No, right?

00:05:37 Speaker 1

You need to know that.

00:05:38 Speaker 1

that mathematics has formalized these arguments.

00:05:42 Speaker 1

It is to get away from language.

00:05:44 Speaker 1

This is more precise.

00:05:46 Speaker 1

It's more accurate and valid in all situations.

00:05:51 Speaker 1

OK.

00:05:52 Speaker 1

Good.

00:05:53 Speaker 1

Fine.

00:05:55 Speaker 1

Right.

00:05:55 Speaker 1

So syllogisms are very basic examples of just valid reasoning, just good logic.

00:06:06 Speaker 1

If you've got an argument which is bad logic, then this is called invalid reasoning.

00:06:12 Speaker 1

OK, so an argument is called valid if the conclusions follow from the premises, and otherwise it is called invalid.

00:06:20 Speaker 1

Invalid if you cannot draw the conclusion from the premises.

00:06:23 Speaker 1

OK?

00:06:33 Speaker 1

You want to try another one?

00:06:35 Speaker 1

Yes, so I'm going to show you another one.

00:06:38 Speaker 1

Let's see.

00:06:39 Speaker 1

Let's accept this.

00:06:40 Speaker 1

Every dog is a bird.

00:06:41 Speaker 1

Yes?

00:06:42 Speaker 1

OK, we accept all dogs are birds.

00:06:45 Speaker 1

We also accept somewhere there exists an elephant that isn't a bird.

00:06:50 Speaker 1

Yes, we accept that.

00:06:51 Speaker 1

Somewhere there's one elephant who's not a bird.

00:06:53 Speaker 1

Right.

00:06:54 Speaker 1

Is this a bit of valid reasoning?

00:06:58 Speaker 1

So every elephant is not a dog.

00:07:04 Speaker 1

I'm asking, is the argument valid?

00:07:07 Speaker 1

So can we draw this conclusion if you accept

00:07:10 Speaker 1

only those premises, that's all.

00:07:12 Speaker 1

You accept those premises, you accept only.

00:07:15 Speaker 1

All dogs are birds, and somewhere there is an elephant who isn't a bird.

00:07:19 Speaker 1

Can you conclude from just that, that every single elephant is not a dog?

00:07:28 Speaker 1

So this elephant that is not a bird, can this particular elephant be a dog?

00:07:35 Speaker 1

No, he is not a dog, because every dog is a bird.

00:07:37 Speaker 1

But what about all the other elephants out there?

00:07:40 Speaker 1

We don't know, right?

00:07:41 Speaker 1

So this is an example of invalid reasoning.

00:07:46 Speaker 1

This argument is invalid.

00:07:48 Speaker 1

Yes, the conclusion does not follow from the premises.

00:07:51 Speaker 1

So you need to try and get away from all your knowledge.

00:07:54 Speaker 1

Yes, we accept only the premises, just accept blindly.

00:07:58 Speaker 1

And then does the conclusion follow, right?

00:08:01 Speaker 1

So this is an invalid argument.

00:08:04 Speaker 1

Okay.

00:08:05 Speaker 1

Let's try another one.

00:08:06 Speaker 1

Another one involves a particular type of bird.

00:08:09 Speaker 1

Have you met this bird before in England?

00:08:12 Speaker 1

Anybody?

00:08:13 Speaker 1

It's a robin.

00:08:14 Speaker 1

Yes, who knew?

00:08:15 Speaker 1

It's very unusual for anyone to know.

00:08:17 Speaker 1

Very good.

00:08:17 Speaker 1

This is a robin.

00:08:18 Speaker 1

Yes, this kind of bird is a robin.

00:08:20 Speaker 1

They are a British bird, and they're very friendly birds.

00:08:24 Speaker 1

And some people will feed them, and then they come every day to your door and ask for some food.

00:08:31 Speaker 1

I think they're also quite aggressive.

00:08:32 Speaker 1

I've seen them fighting each other.

00:08:34 Speaker 1

I don't think they're actually that friendly.

00:08:35 Speaker 1

They're friendly to humans.

00:08:37 Speaker 1

Right.

00:08:38 Speaker 1

So let's try another one.

00:08:41 Speaker 1

Every bird is a fish.

00:08:43 Speaker 1

Let's accept this.

00:08:43 Speaker 1

Yes, if you see a bird, that is actually also a fish.

00:08:47 Speaker 1

There exists somewhere, a robin, that isn't a fish.

00:08:51 Speaker 1

Now, what about the conclusion?

00:08:54 Speaker 1

Is it true?

00:08:55 Speaker 1

Somewhere there exists a robin that's not a bird.

00:09:00 Speaker 1

You all seem to be happy with that.

00:09:01 Speaker 1

So this robin, who is not a fish, can he be a bird, this particular robin?

00:09:05 Speaker 1

No.

00:09:06 Speaker 1

No, he cannot be a bird, because every bird is a fish.

00:09:09 Speaker 1

So this is a valid argument, right?

00:09:12 Speaker 1

So this one is valid.

00:09:14 Speaker 1

OK.

00:09:15 Speaker 1

All right.

00:09:16 Speaker 1

So at this stage, you must be thinking that valid and invalid arguments are not the total answer.

00:09:22 Speaker 1

Because if I read this statement, every elephant is not a dog, that's true.

00:09:29 Speaker 1

But invalid argument.

00:09:30 Speaker 1

Somewhere there exists a robin that's not a bird.

00:09:33 Speaker 1

Well, that's nonsense.

00:09:35 Speaker 1

But it's a valid argument.

00:09:36 Speaker 1

So what's the problem here?

00:09:38 Speaker 1

Just sticking to valid and invalid arguments is not going to give us truth.

00:09:42 Speaker 1

What else do we need?

00:09:45 Speaker 1

The premises must be true, right?

00:09:47 Speaker 1

So the reason why we are getting sort of nonsense from this valid argument is because these premises are not true.

00:09:56 Speaker 1

So when we're talking about

00:09:59 Speaker 1

good arguments, you don't only want valid reasoning, you also want your premises to be true, right?

00:10:06 Speaker 1

The things that you accept.

00:10:10 Speaker 1

If the premises are true and the logic of the argument is valid, then we say that the argument is sound, okay?

00:10:22 Speaker 1

And that is actually what we want.

00:10:24 Speaker 1

We want good reasoning, but also correct premises.

00:10:28 Speaker 1

So we want sound arguments.

00:10:30 Speaker 1

All right, so I'll just over and over again, right?

00:10:53 Speaker 1

What is a valid argument?

00:10:54 Speaker 1

The conclusion follows from the premises.

00:10:56 Speaker 1

What is a sound argument?

00:10:58 Speaker 1

The argument is valid, so conclusion follows from premises and premises are true writers, the two things.

00:11:05 Speaker 1

Okay.

00:11:06 Speaker 1

And we would like, as far as possible, of course, to use sound arguments in all areas, in all aspects of life, right?

00:11:13 Speaker 1

So we try to use sound arguments in all fields of study and in everyday life as well.

00:11:20 Speaker 1

We want to have good logic applied to true premises.

00:11:29 Speaker 1

So that's fine for maybe for science, but in everyday life this can be problematic, right?

00:11:33 Speaker 1

Because there are many issues in everyday life which are not obviously true or obviously false.

00:11:40 Speaker 1

Yes, arguments to do with the criminal justice system, with politics, with human rights, all of these things, it's not obvious always what is true and what is not true.

00:11:54 Speaker 1

OK, so the truth of premises is not always clear in everyday life.

00:12:01 Speaker 1

OK.

00:12:02 Speaker 1

All right.

00:12:03 Speaker 1

Are you following so far?

00:12:05 Speaker 1

Yes, because I'm going to test you.

00:12:07 Speaker 1

Yes?

00:12:07 Speaker 1

All right.

00:12:08 Speaker 1

So on the next page, I'm going to give you 4 arguments.

00:12:13 Speaker 1

And I would like you to discuss with the person or people around you, is the argument valid or invalid?

00:12:21 Speaker 1

And is the argument sound or unsound?

00:12:23 Speaker 1

Yes, and I'll give you five minutes.

00:12:26 Speaker 1

Try and work it out.

00:12:29 Speaker 1

So what does the first one mean, valid or invalid?

00:12:31 Speaker 1

The reason is correct.

00:12:33 Speaker 1

The conclusion follows from the premises.

00:12:35 Speaker 1

And this one means?

00:12:36 Speaker 1

The premises are true.

00:12:38 Speaker 1

Premise is true and logic is good.

00:12:40 Speaker 1

OK.

00:12:41 Speaker 1

All right.

00:12:41 Speaker 1

So let me show you.

00:12:42 Speaker 1

There are four of them.

00:12:45 Speaker 1

Pardon?

00:12:49 Speaker 1

Right, number two, number three, number four.

00:12:52 Speaker 1

Right, you have five minutes, four minutes maybe, to discuss, come to a conclusion.

00:13:01 Speaker 1

Could I start talking again, please?

00:13:10 Speaker 1

Good.

00:13:10 Speaker 1

Let's look over the results.

00:13:12 Speaker 1

So first one, all people are warm-blooded, and all UPC students are warm-blooded.

00:13:18 Speaker 1

If we accept those two, can we conclude that all UPC students are people?

00:13:22 Speaker 1

No.

00:13:24 Speaker 1

No, we cannot.

00:13:25 Speaker 1

Yes, we've got the warm-blooded set, and inside is the people set and the UPC student set.

00:13:29 Speaker 1

So what can you say about this one?

00:13:31 Speaker 1

Valid?

00:13:33 Speaker 1

Invalid.

00:13:34 Speaker 1

Invalid and?

00:13:35 Speaker 1

Unsound yes, it's invalid.

00:13:37 Speaker 1

So this makes it automatically It's invalid so that makes it automatically unsound right invalid and unsound.

00:13:55 Speaker 1

Let's take a look at number two.

00:13:57 Speaker 1

What do you think?

00:13:58 Speaker 1

Is it valid, invalid, sound or unsound?

00:14:03 Speaker 1

Which one do you think?

00:14:07 Speaker 1

Valid.

00:14:07 Speaker 1

Is it sound?

00:14:11 Speaker 1

Yes.

00:14:11 Speaker 1

Is it sound?

00:14:13 Speaker 1

Yes.

00:14:13 Speaker 1

Yes, right.

00:14:14 Speaker 1

So the conclusion does follow from the premises, yes, because some red objects are apples.

00:14:19 Speaker 1

And if it's an apple, it comes from a tree.

00:14:21 Speaker 1

So some red apples come from trees.

00:14:23 Speaker 1

And these premises are true, I think we can say.

00:14:26 Speaker 1

So this argument is sound, yes, and therefore also valid, yes.

00:14:31 Speaker 1

Every sound argument is a valid argument.

00:14:33 Speaker 1

All right, let's go for number three.

00:14:36 Speaker 1

All horses are animals.

00:14:38 Speaker 1

There exists a dog that's not a horse.

00:14:39 Speaker 1

Can we conclude there exists a dog that's not an animal?

00:14:43 Speaker 1

Invalid.

00:14:43 Speaker 1

Invalid.

00:14:44 Speaker 1

So unsound, yes, invalid.

00:14:47 Speaker 1

So this dog that is not a horse, he's not a horse.

00:14:51 Speaker 1

So does that mean he's not an animal?

00:14:53 Speaker 1

No, we cannot conclude it.

00:14:55 Speaker 1

Yes, we know that all horses are animals, but if you're not a horse, you could also be an animal.

00:14:59 Speaker 1

So no, this one's invalid, therefore unsound.

00:15:03 Speaker 1

And what about the last one?

00:15:06 Speaker 1

Valid, but clearly unsound, yes.

00:15:09 Speaker 1

Valid, but unsound.

00:15:10 Speaker 1

OK, good.

00:15:11 Speaker 1

So up to now, I've been talking a little bit about logic and sound arguments, valid arguments, yes.

00:15:18 Speaker 1

Let's return to our original question of what makes something true in mathematics.

00:15:25 Speaker 1

So we've been talking about proof.

00:15:27 Speaker 1

Yes, basically, we use logic to get results in mathematics.

00:15:30 Speaker 1

So to make something true in mathematics, we would like to apply sound arguments.

00:15:36 Speaker 1

That would make us get further results.

00:15:39 Speaker 1

So to obtain true results in mathematics, you want to use sound arguments.

00:15:43 Speaker 1

What does it mean?

00:15:44 Speaker 1

So to make sure that something is true in mathematics, apply a sound argument.

00:15:49 Speaker 1

That means apply some valid reasoning to true premises.

00:15:57 Speaker 1

Hold on a minute.

00:15:59 Speaker 1

That doesn't make sense, does it?

00:16:01 Speaker 1

So how do you know if something's true in mathematics?

00:16:03 Speaker 1

You apply good logic to something that's true in mathematics.

00:16:08 Speaker 1

This is circular, right?

00:16:10 Speaker 1

This is not very good.

00:16:12 Speaker 1

So how are we going to actually know if something is true in mathematics?

00:16:17 Speaker 1

And the answer is maybe less.

00:16:21 Speaker 1

satisfactory than you might think.

00:16:23 Speaker 1

We cannot go around in circles.

00:16:24 Speaker 1

What we do is we accept some things as just being true without proof, right?

00:16:30 Speaker 1

So how do we know premises are true?

00:16:33 Speaker 1

What do we start with?

00:16:35 Speaker 1

And we start with some things that we just accept as being true without proof.

00:16:41 Speaker 1

This is called a set of axioms.

00:16:44 Speaker 1

Okay, so we start with a set of axioms.

00:16:46 Speaker 1

These are statements, we accept them as being true with no proof.

00:17:09 Speaker 1

OK, so as you might imagine, sets of axioms are pretty important.

00:17:13 Speaker 1

We start with a few statements that we accept as being true.

00:17:16 Speaker 1

We apply logic and get more results, yes?

00:17:18 Speaker 1

And then we've got those results.

00:17:19 Speaker 1

We apply more logic, we get more results, yes?

00:17:22 Speaker 1

So this is the idea of the subject of mathematics.

00:17:25 Speaker 1

You can think of mathematics maybe like this.

00:17:28 Speaker 1

We will amend the picture later, yes, but we've got some axioms.

00:17:31 Speaker 1

You apply logical inference.

00:17:33 Speaker 1

This means valid reasoning, yes, valid reasoning, valid arguments.

00:17:39 Speaker 1

You get some more results.

00:17:40 Speaker 1

And then on here you can apply some more valid reasoning and you get some more results.

00:17:45 Speaker 1

You can branch off to the side as well, right?

00:17:48 Speaker 1

This tree could branch off in all directions.

00:17:50 Speaker 1

So it's also based on this set of

00:17:54 Speaker 1

facts that we accept as being true without proof.

00:18:16 Speaker 1

All right.

00:18:17 Speaker 1

And so for the last part of my talk, I would like to talk a bit more about sets of axioms.

00:18:23 Speaker 1

Yes.

00:18:32 Speaker 1

So we have the idea of this sound argument, valid reasoning.

00:18:37 Speaker 1

Now let's look at the axioms that lie sort of at the bottom.

00:18:40 Speaker 1

So sets of axioms.

00:18:42 Speaker 1

So stuff we accept as being true without proof, yes?

00:18:45 Speaker 1

Right, what would we like for a set of things that we just accept?

00:18:50 Speaker 1

And some of the basics, they should be consistent.

00:18:54 Speaker 1

Consistent means that they don't argue with each other, right?

00:18:57 Speaker 1

And they should be not redundant.

00:18:59 Speaker 1

That means we want to accept as small a set as possible.

00:19:02 Speaker 1

We don't want to accept things we don't really need to accept.

00:19:05 Speaker 1

So consistent, no contradictions.

00:19:09 Speaker 1

Not redundant, as few as possible.

00:19:12 Speaker 1

If you can throw one out, then throw it out.

00:19:41 Speaker 1

All right, so given this idea, you might be quite interested.

00:19:46 Speaker 1

What is the set of axioms that lie underneath mathematics?

00:19:49 Speaker 1

Yes, and early logicians spent a long time trying to find this set of axioms which lies underneath mathematics.

00:20:00 Speaker 1

And they were not successful.

00:20:02 Speaker 1

In fact, it was proved by a logician called Godel.

00:20:06 Speaker 1

It is not possible to write down this the set of axioms underpinning all of mathematics.

00:20:15 Speaker 1

Incompleteness theorem.

00:20:26 Speaker 1

He proved something along the lines of, If you've got a set of axioms, you will be able to make a statement that you can neither prove nor disprove with that set of axioms.

00:20:35 Speaker 1

Yes, so your set of axioms is not kind of enough.

00:20:37 Speaker 1

You cannot

00:20:38 Speaker 1

underpin all of mathematics with your set of axioms.

00:20:52 Speaker 1

All right.

00:20:53 Speaker 1

So I would like to now spend the last part of the talk dealing with a particular set of axioms.

00:20:59 Speaker 1

They are axioms that you will have seen at high school to do with geometry, plane geometry.

00:21:04 Speaker 1

You know those problems you've got at high school?

00:21:05 Speaker 1

Find the angle marked X, yes?

00:21:07 Speaker 1

This kind of thing.

00:21:10 Speaker 1

So let's take a look a little bit more at a particular set of axioms.

00:21:15 Speaker 1

And I'd like to look at Euclid's axioms, because it's quite easy to understand.

00:21:22 Speaker 1

All right, so I'm going to list Euclid's axioms.

00:21:25 Speaker 1

So, these are things that we are accepting without...

00:21:27 Speaker 1

proof, right?

00:21:28 Speaker 1

And you are in no way supposed to memorize all this, right?

00:21:33 Speaker 1

The purpose of this now is a demonstration of how different sets of axioms lead to different results.

00:21:41 Speaker 1

You are not expected to memorize or know any of this.

00:21:44 Speaker 1

So it's kind of a demonstration only.

00:21:47 Speaker 1

So what did Euclid's axioms say back in ancient Greece?

00:21:50 Speaker 1

So these were things that we accept without proof.

00:21:53 Speaker 1

He said,

00:21:54 Speaker 1

It's possible to draw a straight line from any point to any other point.

00:21:59 Speaker 1

Can we accept it?

00:22:00 Speaker 1

Yes, people were happy to accept this.

00:22:03 Speaker 1

This is not a problem, right?

00:22:05 Speaker 1

We accept this one.

00:22:07 Speaker 1

It's possible to extend a finite straight line continuously in both directions.

00:22:11 Speaker 1

You can keep going.

00:22:12 Speaker 1

Yes, if you've got a finite line, you should keep going with it.

00:22:20 Speaker 1

OK.

00:22:21 Speaker 1

It's possible to describe a circle with any center and any radius.

00:22:25 Speaker 1

If you know a particular point that is the center and the radius is a certain length, then you can draw a circle.

00:22:32 Speaker 1

Such a thing.

00:22:33 Speaker 1

And then all right angles are equal to one another.

00:22:37 Speaker 1

OK.

00:22:38 Speaker 1

So this was fine.

00:22:39 Speaker 1

People were sort of happy with this one.

00:22:41 Speaker 1

And then he had a fifth axiom.

00:22:44 Speaker 1

I'm going to show it to you now called the parallel postulate.

00:22:47 Speaker 1

The version I have here is not Euclid's original version, but it is an equivalent one.

00:22:53 Speaker 1

It's this one, the parallel postulate.

00:22:55 Speaker 1

It says, given any line and any point that is not on the line, there is exactly one straight line through the point which is parallel to the first line.

00:23:05 Speaker 1

Are we able to draw that?

00:23:09 Speaker 1

Probably, right?

00:23:11 Speaker 1

So given any line, that's a straight line, yes, and given any point,

00:23:18 Speaker 1

It is possible to draw exactly one straight line through the point that's parallel to the first line.

00:23:29 Speaker 1

What does this mean, that's parallel to the first line?

00:23:33 Speaker 1

If I extend it out forever, they will never meet each other.

00:23:36 Speaker 1

Yes, yes, they have the same slope forever.

00:23:39 Speaker 1

They will never meet each other.

00:23:41 Speaker 1

All right.

00:23:42 Speaker 1

And straight away, right from the beginning, when he published his axioms, people were not so happy with this last one.

00:23:49 Speaker 1

It seemed quite a lot more complicated than the first ones.

00:23:54 Speaker 1

And we want our set of axioms to be as small as possible.

00:23:59 Speaker 1

So people are wondering, really, do we need this?

00:24:02 Speaker 1

Does it not follow from the first ones?

00:24:06 Speaker 1

So right from the beginning,

00:24:09 Speaker 1

OK, here I've drawn a picture of it.

00:24:11 Speaker 1

Again, this is what I was trying to draw earlier on.

00:24:15 Speaker 1

But many attempts were made to show that this parallel postulate was a redundant axiom, that we don't need it, that we can throw it out.

00:24:23 Speaker 1

All right, could we go back and think about how would you go about trying to prove that you don't actually need this one?

00:24:36 Speaker 1

There are two ways that you could do it.

00:24:38 Speaker 1

One way would be to try and prove this by using that.

00:24:41 Speaker 1

So you assume this, and you try and get a proof for this.

00:24:44 Speaker 1

Another way to prove that you don't really need this is to suppose that this one is false, and then get a contradiction to one of those.

00:24:54 Speaker 1

That would mean that this one actually must be true, if all of those are true, right?

00:24:58 Speaker 1

So that's kind of one way of trying to prove that you don't really need this parallel postulate.

00:25:03 Speaker 1

So

00:25:05 Speaker 1

One way to prove that we don't need the parallel postulate is to assume that it's false.

00:25:09 Speaker 1

Try and get then a contradiction to the first four axioms.

00:25:12 Speaker 1

So people tried this a lot, and it didn't work.

00:25:15 Speaker 1

They couldn't do it.

00:25:16 Speaker 1

This was unsuccessful.

00:25:24 Speaker 1

Instead, what did they find?

00:25:27 Speaker 1

They assumed that the parallel postulate was false, and

00:25:33 Speaker 1

They tried to get a contradiction to the first four axioms.

00:25:35 Speaker 1

That didn't happen.

00:25:36 Speaker 1

Instead, they just developed more different mathematics.

00:25:41 Speaker 1

So they were developing different mathematics based on the fact that the parallel postulate is false.

00:25:47 Speaker 1

So let's have a look.

00:25:50 Speaker 1

Is the parallel postulate true?

00:25:52 Speaker 1

So originally, the parallel postulate was meant to be modeling the real world, right?

00:25:56 Speaker 1

What do you think?

00:25:57 Speaker 1

Is it true if I've got a line and a point not on the line, I can draw

00:26:03 Speaker 1

another line through the point which is parallel to the first line.

00:26:05 Speaker 1

So I can continue the lines forever, and it goes on, and they never meet.

00:26:10 Speaker 1

Is it true?

00:26:15 Speaker 1

Yes.

00:26:15 Speaker 1

Yeah.

00:26:16 Speaker 1

Is it true?

00:26:18 Speaker 1

Is it true they're trying to model the real world?

00:26:21 Speaker 1

Is it true in the flat world, if the world is flat forever?

00:26:25 Speaker 1

Yes, right.

00:26:26 Speaker 1

If the world is just going to go flat forever, then yes, you can extend the lines forever and they will never meet.

00:26:32 Speaker 1

This is true.

00:26:33 Speaker 1

Do we live in the flat world?

00:26:36 Speaker 1

In some sense, no.

00:26:36 Speaker 1

But in some sense also, yes.

00:26:38 Speaker 1

I mean, living in the flat world, yes.

00:26:40 Speaker 1

If I want to build a bridge between this building and, you know, main building at UCL,

00:26:47 Speaker 1

It's a flat world, right?

00:26:49 Speaker 1

It's flat.

00:26:50 Speaker 1

So, in some sense, the world is kind of flat, but overall, our world, of course, is not flat.

00:26:55 Speaker 1

It's a very cool world.

00:26:56 Speaker 1

So, what happens if we extend straight lines forever?

00:27:00 Speaker 1

Like, where do they go?

00:27:01 Speaker 1

Like, they go out to Bedford Way, Euston Station, Camden, and then where are they going?

00:27:07 Speaker 1

To America, Canada?

00:27:09 Speaker 1

Where do the lines go?

00:27:10 Speaker 1

Do they wrap around the world?

00:27:11 Speaker 1

Do they go into space?

00:27:13 Speaker 1

It's not so clear, yes?

00:27:14 Speaker 1

So,

00:27:15 Speaker 1

So our idea of extend forever is based on this, yes.

00:27:19 Speaker 1

Are they going around the world?

00:27:20 Speaker 1

Who knows?

00:27:22 Speaker 1

And then, so that's just our world.

00:27:23 Speaker 1

We live over here, but we also live in an expanding universe, which is exploding.

00:27:28 Speaker 1

I don't understand the picture at all.

00:27:30 Speaker 1

I just found it on Google.

00:27:32 Speaker 1

But the universe is expanding, right?

00:27:34 Speaker 1

So what does it really mean to extend lines forever?

00:27:41 Speaker 1

Right.

00:27:42 Speaker 1

But I think we can agree that in a flat world, the parallel postulate is true.

00:27:47 Speaker 1

And in a flat world, which is the world of a piece of paper, all five of Euclid's axioms are true.

00:27:55 Speaker 1

And this is the material you were studying in high school, right?

00:27:59 Speaker 1

This is called plane geometry, the geometry of the plane, OK?

00:28:04 Speaker 1

The two-dimensional world.

00:28:05 Speaker 1

So Euclidean geometry.

00:28:08 Speaker 1

Based on all five of Euclid's axioms is also called plane geometry.

00:28:13 Speaker 1

This is the stuff that you studied at school, right?

00:28:15 Speaker 1

So at school, yes, if you have a line and you've got a point, you can construct a parallel line to the first line.

00:28:23 Speaker 1

Yes?

00:28:23 Speaker 1

Remember it?

00:28:25 Speaker 1

Do you want reminding?

00:28:27 Speaker 1

Come on, then.

00:28:31 Speaker 1

Find angle BDF.

00:28:44 Speaker 1

Oh, he's the middle of the circle, I think we can assume, yeah.

00:28:47 Speaker 1

BDF.

00:28:52 Speaker 1

Yes, BDF.

00:28:53 Speaker 1

BDF.

00:28:55 Speaker 1

What's this angle here where my pointer is?

00:28:57 Speaker 1

26.

00:28:59 Speaker 1

What's this angle here is?

00:29:00 Speaker 1

26.

00:29:01 Speaker 1

26.

00:29:02 Speaker 1

You're right.

00:29:02 Speaker 1

What is that friend next to it here?

00:29:04 Speaker 1

What's this one?

00:29:08 Speaker 1

What is it?

00:29:10 Speaker 1

I didn't hear you, so I don't say it's wrong.

00:29:12 Speaker 1

I'll ask, what is it?

00:29:15 Speaker 1

The whole thing is 39, yes, the whole thing is 39, right?

00:29:18 Speaker 1

So this is Euclidean geometry, right?

00:29:20 Speaker 1

So you are using all Euclid's axioms to...

00:29:24 Speaker 1

So the whole angle BDF from here to here to here.

00:29:29 Speaker 1

This is 13, yes, this one is 13, because this angle here is 90 degrees.

00:29:34 Speaker 1

So this is 13, and this is the same size, because these two have the same length.

00:29:40 Speaker 1

OK, fine.

00:29:41 Speaker 1

So Euclidean geometry based on all Euclid's axioms.

00:29:45 Speaker 1

OK, now suppose we now let's look at what mathematics would look like if we assume that the parallel postulate is false.

00:29:54 Speaker 1

OK, now there are two ways that we could assume that the parallel postulate is false.

00:30:00 Speaker 1

One way is, and it can be proved that you must assume one or the other.

00:30:04 Speaker 1

Yes, one

00:30:07 Speaker 1

way that it could be false is this.

00:30:09 Speaker 1

Given any point and any line, no straight line parallel to the first line can be drawn through the point.

00:30:16 Speaker 1

So if you've got a line, you've got a point, you cannot draw a line through the point, which, when you continue it forever, will never meet the first line.

00:30:32 Speaker 1

Or you could assume

00:30:34 Speaker 1

Given any point and any line, you can draw more than one line through the point, which is parallel to the first line.

00:30:42 Speaker 1

There is more than one that you can draw and extend forever and that will never meet.

00:30:47 Speaker 1

OK?

00:30:48 Speaker 1

It can be shown that you should assume either this or that you should assume this.

00:30:52 Speaker 1

So I would like to look at what geometry could look like given this first situation.

00:30:57 Speaker 1

OK?

00:30:58 Speaker 1

So remember, people were trying to prove the parallel postulate is false by assuming--so

00:31:04 Speaker 1

Trying to-- by assuming it's false and getting a contradiction.

00:31:07 Speaker 1

This didn't happen.

00:31:08 Speaker 1

So let's see what happens if we do assume that this is false.

00:31:13 Speaker 1

So I would like to look now at what does maths look like if we assume this?

00:31:19 Speaker 1

Given any point and any line, you cannot draw a straight line parallel to the first line through the point.

00:31:28 Speaker 1

Yes, this means infinite lines.

00:31:34 Speaker 1

So I'm going to do some maths.

00:31:36 Speaker 1

It's not rigorous maths, OK?

00:31:39 Speaker 1

I'm going to assume some symmetry, and it's just a demonstration.

00:31:43 Speaker 1

It's not very rigorous mathematics.

00:31:46 Speaker 1

So I'm assuming symmetry.

00:31:47 Speaker 1

And also, you are definitely not going to get tested on this.

00:31:51 Speaker 1

It is an example to show you that maths will look very different if the axioms you assume are very different, right?

00:31:59 Speaker 1

So if you assume all Euclid's axioms, you will get the high school geometry.

00:32:04 Speaker 1

you don't assume the parallel postulate and you assume this in its place, let's see what kind of strange mathematics you will get.

00:32:10 Speaker 1

Yes, and it will be different.

00:32:12 Speaker 1

It's not rigorous.

00:32:15 Speaker 1

Okay, what can you tell me then, let's assume this, okay?

00:32:19 Speaker 1

What can you tell me then if you draw any two lines?

00:32:23 Speaker 1

What must eventually happen if you extend them out?

00:32:27 Speaker 1

They would have to?

00:32:29 Speaker 1

They would have to make an angle, they would have to eventually?

00:32:33 Speaker 1

Meet.

00:32:34 Speaker 1

They would have to be meet.

00:32:35 Speaker 1

They cannot be parallel, yes?

00:32:37 Speaker 1

So if I draw any two lines, they will eventually meet each other.

00:32:41 Speaker 1

Otherwise, this is false, yes?

00:32:43 Speaker 1

Otherwise, you've got a line, you've got a point, and you've got a parallel line, and you extend forever, parallel forever.

00:32:49 Speaker 1

So any two lines must eventually meet, OK?

00:32:53 Speaker 1

If we assume this.

00:32:55 Speaker 1

OK.

00:32:57 Speaker 1

Right, so I'm going to try and draw.

00:33:00 Speaker 1

Now, I'm drawing on this screen, which is a plane, yes?

00:33:03 Speaker 1

So I cannot really draw correctly.

00:33:07 Speaker 1

When I draw a line, if I draw a straight line, it will have to be straight.

00:33:12 Speaker 1

But we must just imagine my straight lines I'm going to draw are going to be crooked in a minute, yes, because they have to be.

00:33:19 Speaker 1

So

00:33:20 Speaker 1

What do I want to do?

00:33:22 Speaker 1

I want to start with any straight line in my geometry with two points on it.

00:33:28 Speaker 1

So in this geometry, I take a straight line with two points on it.

00:33:33 Speaker 1

And I want to see what happens when I extend this straight line infinitely in both directions.

00:33:40 Speaker 1

So to start with, can we draw a perpendicular line here?

00:33:46 Speaker 1

and a perpendicular line here, yes, 90 degree lines, two 90 degree lines going up.

00:33:52 Speaker 1

And I extend them.

00:33:53 Speaker 1

What must happen when I extend them?

00:33:56 Speaker 1

They must intersect, yes.

00:33:57 Speaker 1

So I'm going to draw that.

00:33:59 Speaker 1

Now I cannot draw them straight lines because they will just go up and not intersect.

00:34:02 Speaker 1

So I will draw them skew, yes.

00:34:06 Speaker 1

So I construct perpendicular lines at A and B.

00:34:08 Speaker 1

So at 90 degrees here, at 90 degrees here, and I extend them up, they must meet.

00:34:13 Speaker 1

They're going to meet at O.

00:34:15 Speaker 1

And I'm assuming some symmetry.

00:34:17 Speaker 1

So I will assume that this and this now have the same length.

00:34:22 Speaker 1

My geometry doesn't favor one side or the other side.

00:34:26 Speaker 1

So I have straight lines and straight lines, and I'm meeting at point O, OK?

00:34:31 Speaker 1

All right.

00:34:32 Speaker 1

Straight away, we have something odd, isn't it?

00:34:34 Speaker 1

What are the angles in this triangle?

00:34:37 Speaker 1

More than 180, yes?

00:34:39 Speaker 1

So straight away, more than 180.

00:34:41 Speaker 1

What I'm very interested in is the size of this angle.

00:34:44 Speaker 1

I want to show that I can make the size of this angle anything that I want.

00:34:49 Speaker 1

OK?

00:34:49 Speaker 1

So this is my aim.

00:34:50 Speaker 1

I want to show that I can make it smaller or I can make it bigger, the size of angle AOB.

00:34:57 Speaker 1

OK?

00:34:58 Speaker 1

So let's see what would happen if I went halfway between A and B and I drew another perpendicular going up.

00:35:06 Speaker 1

So I draw another perpendicular line going up.

00:35:08 Speaker 1

It must intersect lines AO and BO.

00:35:12 Speaker 1

Yes, it must intersect both of them.

00:35:14 Speaker 1

And by symmetry, it will cut through at point O.

00:35:17 Speaker 1

Okay.

00:35:19 Speaker 1

So here's angle A and there's point A and point B.

00:35:22 Speaker 1

I'll go halfway between them.

00:35:23 Speaker 1

Q, I'll go up.

00:35:25 Speaker 1

We accept some symmetry.

00:35:26 Speaker 1

It's going to cut through point O.

00:35:28 Speaker 1

So can you see what happened to the angle at O?

00:35:31 Speaker 1

I am able to make it smaller, right?

00:35:34 Speaker 1

I'm able to halve it.

00:35:35 Speaker 1

OK, so I'm able to halve it.

00:35:37 Speaker 1

I could repeat the performance.

00:35:38 Speaker 1

If I took another point here and another point here and drew lines going up, they would also cut through O, right?

00:35:46 Speaker 1

So I think I have a slide with that.

00:35:48 Speaker 1

Not very good, but yes.

00:35:51 Speaker 1

go up again, right?

00:35:52 Speaker 1

So I've made the angles up here even smaller.

00:35:55 Speaker 1

I've made them all a quarter of the size, okay?

00:35:58 Speaker 1

So I can make smaller and smaller and smaller and smaller.

00:36:00 Speaker 1

Can I make this angle at the top bigger?

00:36:03 Speaker 1

Sure, right?

00:36:06 Speaker 1

If I go from A to B, and now I take another point further along, point C, further along here, and make a line going up,

00:36:20 Speaker 1

Yes, so there was my A and my B.

00:36:22 Speaker 1

Now I take point C and make a line going up at 90 degrees, and it will also intersect there, yes, again by symmetry.

00:36:30 Speaker 1

So I've made this one really big, yes, and now I could make this smaller.

00:36:35 Speaker 1

So all you're supposed to see is that this angle here, I can make it into whatever size I want.

00:36:41 Speaker 1

I can make it bigger, I can make it smaller.

00:36:43 Speaker 1

Okay, so given that,

00:36:48 Speaker 1

I'm now going to choose a point on the line to make that angle at the top equal to 90 degrees.

00:36:55 Speaker 1

So I'll start with my point A, which was on the line, move along until I find point G.

00:37:00 Speaker 1

And now I want to organize it so that it's 90 degrees up at the top there.

00:37:04 Speaker 1

OK.

00:37:06 Speaker 1

Good.

00:37:08 Speaker 1

What do the angles in that triangle add up to?

00:37:11 Speaker 1

270.

00:37:12 Speaker 1

270, yes.

00:37:12 Speaker 1

OK.

00:37:13 Speaker 1

All right.

00:37:14 Speaker 1

And then I can continue this.

00:37:16 Speaker 1

I'm now going to move on from G and put another point here and construct a perpendicular up so that I get another 90-degree angle.

00:37:28 Speaker 1

OK.

00:37:28 Speaker 1

So let's see.

00:37:29 Speaker 1

Can we follow?

00:37:31 Speaker 1

So here was A.

00:37:32 Speaker 1

This was my original G.

00:37:33 Speaker 1

This was 90 degrees.

00:37:35 Speaker 1

Then I measure off this length-- here it is again--

00:37:39 Speaker 1

construct another perpendicular, and so I'll have another 90, and again another 90, and again another 90.

00:37:46 Speaker 1

Okay, so looking like this.

00:37:49 Speaker 1

Okay, all right.

00:37:54 Speaker 1

So I've got all those 90 degrees up there at point O, but point O actually exists.

00:38:01 Speaker 1

Yes.

00:38:01 Speaker 1

It might be an expanding universe or something, but point O actually exists.

00:38:05 Speaker 1

So the angles around point O are now adding up to 360, and point O actually exists somewhere.

00:38:13 Speaker 1

Yes.

00:38:14 Speaker 1

So maybe up to now, my original line I've drawn straight, and my other straight lines I've drawn wonky.

00:38:22 Speaker 1

But maybe if I draw my original line like this,

00:38:28 Speaker 1

This point actually exists.

00:38:30 Speaker 1

So what can you say must actually be happening?

00:38:34 Speaker 1

Point O actually exists, and these angles are 360 degrees.

00:38:37 Speaker 1

So actually, this line and this line must be the same, yes, because point O is existing somewhere.

00:38:46 Speaker 1

And we've got 360 degrees around point O.

00:38:49 Speaker 1

So what is really happening is this, yes, that line O

00:38:56 Speaker 1

G triple prime and OA must actually be the same line.

00:38:59 Speaker 1

Yes?

00:39:00 Speaker 1

And my line must look like this.

00:39:03 Speaker 1

So this is now my original line.

00:39:06 Speaker 1

OK?

00:39:06 Speaker 1

It was the original line I took.

00:39:08 Speaker 1

It had point A on it.

00:39:09 Speaker 1

It had point B on it.

00:39:10 Speaker 1

Point B was living somewhere here.

00:39:12 Speaker 1

We've thrown away point B.

00:39:14 Speaker 1

Yes?

00:39:14 Speaker 1

So it looks like this.

00:39:16 Speaker 1

So what do we see in my new geometry?

00:39:20 Speaker 1

I see that

00:39:21 Speaker 1

Any straight line will wrap back onto itself in this geometry.

00:39:25 Speaker 1

If you extend far enough, it will wrap back onto itself.

00:39:33 Speaker 1

All right, so what do we see?

00:39:35 Speaker 1

If we accept, instead of Euclid's parallel postulate, a different axiom, then your geometry is quite different.

00:39:44 Speaker 1

Your straight lines are going to wrap around and be circles.

00:39:48 Speaker 1

Angles in a triangle could add up to more than 180 degrees.

00:39:51 Speaker 1

So you have completely different geometry.

00:39:54 Speaker 1

OK.

00:39:56 Speaker 1

Fine.

00:39:57 Speaker 1

So what really do we mean by something is true in mathematics?

00:40:02 Speaker 1

I mean, what is true?

00:40:04 Speaker 1

Do the angles in a triangle add up to 180, or can they add up to more?

00:40:10 Speaker 1

And our picture of mathematics actually should look like this.

00:40:14 Speaker 1

It depends what set of axioms you're using.

00:40:17 Speaker 1

Yes.

00:40:18 Speaker 1

So if you've got some particular axioms, like all of Euclid's five axioms, you will get some results.

00:40:25 Speaker 1

And you can get more results, more results.

00:40:27 Speaker 1

You can keep going.

00:40:28 Speaker 1

If you have got completely different axioms, yes, like we replaced the parallel postulate with the thing that we had, that there are no parallel lines, then you get completely different results, and you could keep going, and you can get different further results, and so on.

00:40:46 Speaker 1

So what does it mean if something's true in mathematics?

00:40:50 Speaker 1

It means that you've used good logic, and the truth depends very much on the sets of axioms that you started with.

00:41:00 Speaker 1

There isn't actually a truth.

00:41:03 Speaker 1

It is truth dependent on the axioms.

00:41:07 Speaker 1

So in mathematics, different sets of axioms lead to different theories.

00:41:13 Speaker 1

The truth of any result depends on the axioms that you originally assume.

00:41:19 Speaker 1

There isn't a truth.

00:41:22 Speaker 1

It depends upon what axioms you were originally working with.

00:41:29 Speaker 1

Good.

00:41:31 Speaker 1

Fine.

00:41:32 Speaker 1

So what does it mean if we extend a straight line infinitely in both directions in the real world?

00:41:36 Speaker 1

I mean, I'll go out into Bedford Way, Camden.

00:41:41 Speaker 1

Ireland, Canada, where will it go?

00:41:42 Speaker 1

Will it wrap around the world?

00:41:44 Speaker 1

Is that what we showed?

00:41:46 Speaker 1

No, that's not what we showed at all, right?

00:41:48 Speaker 1

So we must be very careful.

00:41:49 Speaker 1

Mathematics is we accept some axioms, we apply logic, we get results.

00:41:54 Speaker 1

Does that mean we are modeling the real world?

00:41:57 Speaker 1

No, right?

00:41:58 Speaker 1

It is mathematics.

00:41:59 Speaker 1

It's on a piece of paper.

00:42:00 Speaker 1

It doesn't mean the real world magically does that, yes?

00:42:04 Speaker 1

So

00:42:06 Speaker 1

Einstein showed that the universe is non-Euclidean.

00:42:09 Speaker 1

The universe does not obey the parallel postulate.

00:42:14 Speaker 1

But he was very careful about this.

00:42:16 Speaker 1

He says, as far as the propositions of mathematics refer to reality, they are not certain.

00:42:22 Speaker 1

So if you do something in maths, it doesn't mean this is in physics, in the real world.

00:42:27 Speaker 1

Not at all, yes?

00:42:28 Speaker 1

And as far as they are certain, they do not refer to reality.

00:42:33 Speaker 1

Be quite careful.

00:42:34 Speaker 1

There's a difference between maths and modeling the real world physics, maybe.

00:42:40 Speaker 1

OK.

00:42:41 Speaker 1

Good.

00:42:41 Speaker 1

That's all.

00:42:42 Speaker 1

I'm done.

00:42:42 Speaker 1

Thank you very much.